S-Duality from OM-Theory

Teruhiko Kawano and Seiji Terashima

Department of Physics, University of Tokyo

Hongo, Tokyo 113-0033, Japan

kawano@hep-th.phys.s.u-tokyo.ac.jp
seiji@hep-th.phys.s.u-tokyo.ac.jp

We study the compactification of OM-theory on tori and show a simple heuristic derivation of the S-duals of noncommutative open string theory in diverse dimensions from the OM-theoretical point of view. In particular, we identify the S-duality between noncommutative open string theory and noncommutative Yang-Mills theory in $(3 + 1)$ dimensions as the exchange of two circles of a torus on which OM-theory is compactified. Also, we briefly discuss T-duality between noncommutative open string theories.

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1. Introduction

Recently, in the papers [1,2], the authors found a new open string theory in which there were no closed strings. Let us consider a D-brane in a flat Minkowski space with a background B-field. We can see that open strings ending on the D-brane couple to the B-field, but that closed strings do not. When the temporal components of the background B-field are turned on, since strings spatially extend in one dimension, the B-field in the worldsheet action can contribute to the effective tension of open strings and can even make the tension vanish, if we tune the value of the temporal components appropriately. In the limit that the string tension goes to infinity (*i.e.*, $\alpha' \to 0$), we can keep the effective tension of open strings fixed, if the value of the B-field is arranged to be critical. Therefore, gravity decouples from the open string sector in the limit, where we still have all the exciting modes of the open strings. This open string theory is called noncommutative open string (NCOS) theory. Related topics have also been discussed in [3-10].

Let us explain the NCOS limit in some detail. Suppose that a Dp-brane is in a p-dimensional flat space with a background B-field and a metric

$$g_{\mu\nu} = \eta_{\mu\nu} \quad (\mu, \nu = 0, 1), \qquad g_{ij} = f^2 \delta_{ij} \quad (i, j = 2, \ldots, p), \tag{1.1}$$

and $f \to 0$ in the NCOS limit. By using a gauge symmetry, under which the gauge field A on the D-brane and the B-field B transform as $A \to A - \Lambda$, $B \to B + d\Lambda$, we make the background B-field vanish and, instead, obtain the corresponding background field strength $F = dA$. In order to take the field strength to the critical value in the limit, we set

$$2\pi\alpha' F_{01} = 1 - \frac{1}{2} f^2. \tag{1.2}$$

Translating these parameters into the open string moduli in [11], we find that the noncommutative parameter Θ^{01} becomes $2\pi\alpha'_{\text{eff}}$, where α'_{eff} is related to f as $f^2 = (\alpha'/\alpha'_{\text{eff}})$. Since the open string metric G_{AB} $(A, B = 0, \ldots, p)$ equals $f^2\eta_{AB}$, the open string coupling constant G_o^2, defined as $g_{\text{str}}\sqrt{-\det G}/\sqrt{-\det(g + 2\pi\alpha' F)}$, turns out to be $g_{\text{str}}f$. The NCOS limit is defined as $\alpha' \to 0$ ($f \to 0$), $g_{\text{str}} \to \infty$ with α'_{eff} and G_o^2 fixed. In the limit, the effective string tension becomes $1/(2\pi\alpha'_{\text{eff}})$, which gives the scale of the exciting modes of open strings.

In the paper [12], it was proposed that OM-theory describes the strong coupling limit of $(4+1)$-dimensional NCOS theory (NCOS$_{4+1}$). The relation of OM-theory to NCOS$_{4+1}$ is similar to that of M-theory to type IIA superstring theory. The effective tension of open

1

membranes within M5-branes with nonvanishing temporal components of the background three-form tensor field can be different from the tension $M_p^3/(2\pi)^2$ of membranes where M_p is the eleven-dimensional gravitational scale. In the limit $M_p \to \infty$, gravity would decouple from the open membranes, if the effective tension is kept finite. In this way, OM-theory is defined as the limit of this system.

In order to elucidate OM-theory more precisely, we begin with a single M5-brane in a flat Minkowski spacetime where we have the metric (1.1) with $p = 5$ and a background worldvolume three-form field strength H

$$H_{012} = M_p^3 \tanh\beta. \tag{1.3}$$

Here $f^2 = 2\exp(-\beta)/\cosh\beta = 2M_{\text{eff}}^3/M_p^3$, where $M_{\text{eff}}^3/(2\pi)^2$ is the effective tension of open membranes in the OM limit. The self-duality of the field H means, as discussed in [13], that

$$H_{345} = M_{\text{eff}}^3 f \sinh\beta. \tag{1.4}$$

Now we can define OM-theory by the limit $M_p \to \infty$ with M_{eff} fixed. In this limit, open membranes (M2-branes) stretched on the x^1-x^2 plane or on the x^i-x^j plane ($i, j = 3, 4, 5$) have a finite effective tension.

NCOS$_{4+1}$ can be obtained by the compactification of OM-theory on S^1 where the coordinate x^2 is periodically identified and the radius is R_2. As we will see in the next section, by exploiting the relation between an M5-brane wrapped on S^1 in M-theory and a D4-brane in type IIA superstring theory, the open string moduli is given in terms of the parameters R_2, M_{eff} as $1/\alpha'_{\text{eff}} = 2R_2 M_{\text{eff}}^3$ and $G_o^2 = \sqrt{2}(R_2 M_{\text{eff}})^{\frac{3}{2}}$. Thus, the strong coupling limit $G_o \to \infty$ of NCOS$_{4+1}$ can be seen to be the decompactification (i.e., $R_2 \to \infty$) of the OM-theory on S^1, and NCOS$_{4+1}$ becomes (5+1)-dimensional OM-theory in the limit.

In [2], the strong coupling limit of (3+1)-dimensional NCOS theory has also been discussed. Since NCOS$_{3+1}$ is given by the NCOS limit of a D3-brane in type IIB superstring theory, by S-duality of type IIB superstring theory we find that the strong coupling behavior of NCOS$_{3+1}$ is described by (3+1)-dimensional spatially noncommutative Yang-Mills theory.

Aspinwall [14] and Schwarz [15] interpreted S-duality of the nine-dimensional type IIB superstring theory as the modular transformation of a torus on which M-theory is

compactified; see also [16]. This is one of many examples where M-theory plays an important role to unify superstring theories and eleven-dimensional supergravity. In this paper, by applying their idea to OM-theory compactified on a two-dimensional torus, we give a simple derivation of S-duality between spatially noncommutative Yang-Mills theory and NCOS theory in (3+1) dimensions. To this end, we show that the same OM limit in this case can be identified as both of the NCOS limit and the NCYM limit. Then, as Aspinwall and Schwarz gave the closed string coupling in terms of the moduli of the torus, we obtain the relation between the *open* string couplings in NCYM and NCOS by making use of the underlying torus.

Furthermore, we also discuss from the OM-theoretical point of view, the compactification of OM-theory on higher dimensional tori to give a heuristic derivation of the S-duals of NCOS theories in diverse dimensions. The authors of the paper [12] have already identified the S-dualized theories. NCOS theory in (1+1) dimensions has been discussed in more detail in [8]. We use an M2-brane to discuss the S-duality of this theory in section 3. We also discuss T-duality between NCOS theories in section 4.

2. S-duality from OM-theory

2.1. Four-dimensional S-duality from OM-theory

In this subsection, we consider OM-theory compactified on a rectangular torus, where the coordinates x^2 and x^3 are periodically identified as $x^2 \sim x^2 + 2\pi R_2$ and $x^3 \sim x^3 + 2\pi L_3$, and their radii are R_2 and $R_3 = fL_3$, respectively. Note that radius R_3 is much smaller than R_2. The compactification of M-theory on a circle of radius R gives type IIA superstring theory with the string coupling constant $g_{\text{str}} = (RM_{\text{p}})^{\frac{3}{2}}$ and the string length l_{s} given by $l_{\text{s}}^2 = \alpha' = (1/RM_{\text{p}}^3)$. M5-branes wrapped on this circle become D4-branes in type IIA theory. Since we have two circles in the x^2 and x^3 directions in the torus, two IIA theories can be obtained by deciding which circle to identify as the above circle of radius R. T-duality then maps the two IIA theories into two corresponding IIB theories in nine dimensions. These IIB theories are found to be S-dual to each other, as Aspinwall and Schwarz have shown in [14,15,16].

First, identifying R_2 as R, we regard the wrapped M5-brane as a D4-brane in IIA theory with the string coupling constant $g_{\text{str}}^{(2)} = (R_2 M_{\text{p}})^{\frac{3}{2}}$ and the string tension

$$\frac{1}{\alpha'_2} = R_2 M_{\text{p}}^3 = 2f^{-2} R_2 M_{\text{eff}}^3 \tag{2.1}$$

3

on the circle of the radius $R_3 = fL_3$. The field strength of the gauge field on the D4-brane worldvolume has a nearly critical electric component

$$F_{01} = \frac{1}{2\pi} R_2 H_{012} = \frac{1}{2\pi\alpha'_2} \tanh\beta \sim \frac{1}{2\pi\alpha'_2} \left(1 - \frac{1}{2} f^2 \right). \tag{2.2}$$

If we take the limit $R_3 \to \infty$, this theory turns into $(4+1)$-dimensional NCOS theory with the effective string tension

$$\frac{1}{\alpha'^{(2)}_{\text{eff}}} = 2R_2 M_{\text{eff}}^3 \tag{2.3}$$

and the open string coupling constant $G_{\text{o}}^2 = \sqrt{2}(R_2 M_{\text{eff}})^{\frac{3}{2}}$, as shown in [12].

Under the T-duality on the circle of the finite radius R_3, the D4-brane is mapped to a D3-brane in type IIB theory compactified on a circle of radius

$$\widetilde{R}_3 = \frac{\alpha'_2}{R_3} = \frac{f}{2M_{\text{eff}}^3 R_2 L_3}, \tag{2.4}$$

where the string coupling constant is

$$\tilde{g}_{\text{str}}^{(2)} = g_{\text{str}}^{(2)} \left(\frac{\widetilde{R}_3}{R_3} \right)^{\frac{1}{2}} = \frac{R_2}{f\, L_3}. \tag{2.5}$$

The open string metric G_{AB} $(A, B = 0, 1, 4, 5)$ and the noncommutative parameter Θ^{AB} on the D3-brane are given as in [11] by $G_{AB} = f^2 \eta_{AB}$ and $\Theta^{01} = 2\pi\alpha'^{(2)}_{\text{eff}}$. The open string coupling constant turns out to be

$$\widetilde{G}_{\text{E}}^2 = \frac{R_2}{L_3}. \tag{2.6}$$

Thus, we can see that the NCOS limit in NCOS$_{3+1}$ agrees with the OM limit with R_2 and L_3 fixed. Note that, since the mass of D-strings wound once on the circle is

$$M_{\text{D1}} \sim \frac{1}{\tilde{g}_{\text{str}}^{(2)}} \frac{\widetilde{R}_3}{\alpha'_2} = \frac{1}{R_2} \tag{2.7}$$

and remains finite in the OM limit (*i.e.*, $M_{\text{p}} \to \infty$ with M_{eff} fixed), the D-strings are not decoupled from the worldvolume theory, although the tension of D-strings is very large: $T_{\text{D1}} \sim 1/(f\widetilde{G}_{\text{E}}^2 \alpha'^{(2)}_{\text{eff}}) \gg 1/\alpha'^{(2)}_{\text{eff}}$. Thus, to discard those additional degrees of freedom and obtain $(3+1)$-dimensional NCOS theory, after taking the OM limit we should take the limit $R_2 \to 0$. In order to keep the effective string tension $1/\alpha'^{(2)}_{\text{eff}}$ and the open string coupling constant $\widetilde{G}_{\text{E}}$ fixed in the limit, we have to take the limit $M_{\text{eff}}^3 \to \infty$ and $L_3 \to 0$ at the same time.

Next, let us think of the M5-brane on the torus as a D4-brane in another type IIA theory with the string coupling constant $g_{\text{str}}^{(3)} = (R_3 M_{\text{p}})^{\frac{3}{2}}$ and

$$\alpha'_3 = \frac{f}{2L_3 M_{\text{eff}}^3} \tag{2.8}$$

on the circle of the radius R_2. In this picture, the field strength on the worldvolume has a nonzero magnetic component

$$2\pi F_{45} = R_3 H_{345} \sim 2L_3 M_{\text{eff}}^3. \tag{2.9}$$

By T-duality on the circle in the x^2 direction, we obtain a D3-brane in type IIB theory compactified on a circle of radius

$$\widetilde{R}_2 = \frac{\alpha'_3}{R_2} = \frac{f}{2R_2 L_3 M_{\text{eff}}^3}, \tag{2.10}$$

with the string coupling constant

$$\tilde{g}_{\text{str}}^{(3)} = g_{\text{str}}^{(3)} \left(\frac{R'_2}{R_2} \right)^{\frac{1}{2}} = f \frac{L_3}{R_2} = \frac{1}{\tilde{g}_{\text{str}}^{(2)}}. \tag{2.11}$$

The open string coupling constant becomes

$$\widetilde{G}_{\text{M}}^2 = \tilde{g}_{\text{str}}^{(3)} \sqrt{\frac{\det G}{\det(g + 2\pi\alpha'_2 F)}} = \frac{L_3}{R_2}. \tag{2.12}$$

The open string metric and the noncommutative parameter are given by $G_{MN} = \eta_{MN}$ ($M, N = 0, 1, 4, 5$) and

$$\Theta^{45} = \frac{\pi}{L_3 M_{\text{eff}}^3}. \tag{2.13}$$

From (1.1), (2.8), and (2.9), we can verify that the OM limit with R_2 and L_3 fixed is the same limit as the NCYM limit found in [11,12]. Note that, in this limit, the string coupling constant $\tilde{g}_{\text{str}}^{(3)}$ goes to zero. Therefore, the perturbative IIB string theory gives a good description in the limit. However, as the wound D-strings in the previous case are S-dual to open strings wound on the circle in the x^2 direction, the mass of the open strings is $M_{\text{F1}} \sim R'_2/\alpha'_2 = 1/R_2$, and we obtain extra degrees of freedom from the wound open strings in the limit, besides those in (3+1)-dimensional noncommutative Yang-Mills theory (NCYM). Therefore, to decouple these degrees of freedom, we need to take R_2 to zero. In this limit, since we want to keep $\widetilde{G}_{\text{M}}$ and Θ^{45} fixed, we have to take the limit

5

$L_3 \to 0$ and $M_{\text{eff}} \to \infty$. Thus, the theory on the D3-brane becomes $(3+1)$-dimensional noncommutative Yang-Mills theory (NCYM) in the above limit.

This limit is in agreement with that in the previous case used to obtain NCOS$_{3+1}$. Comparing the open string coupling constant $\widetilde{G}_{\text{E}}$ in (2.6) with $\widetilde{G}_{\text{M}}$ in (2.12) shows that

$$\widetilde{G}_{\text{E}} = \frac{1}{\widetilde{G}_{\text{M}}} \tag{2.14}$$

and that the exchange $R_2 \leftrightarrow L_3$ corresponds to S-duality between NCOS theory and NCYM theory in $(3+1)$ dimensions. We also notice that the fluxes of the electric field F_{01} and the magnetic field F_{45} come from the same background self-dual H_{012} and are indeed unified in OM-theory, although they seem very different in lower dimensions.

In the NCYM$_{3+1}$, as a consistency check, let us consider the commutative limit where $\Theta^{45} \to 0$ with the open string coupling $\widetilde{G}_{\text{M}}$ fixed. In this limit, the NCYM becomes commutative $\mathcal{N} = 4$ Yang-Mills theory in $(3+1)$ dimensions, which shows the well-known electric-magnetic duality. On the other hand, we can consider the corresponding limit in the NCOS$_{3+1}$. Since, as is seen from (2.3), (2.12), and (2.13), we have the relation $\Theta^{01} = \widetilde{G}_{\text{M}}^2 \Theta^{45}$, the commutative limit corresponds to the limit $\Theta^{01} \to 0$ with the open string coupling $\widetilde{G}_{\text{E}}$ fixed. By the relation $\Theta^{01} = 2\pi \alpha'^{(2)}_{\text{eff}}$, we can see that all the massive modes of open strings in NCOS theory decouple in this limit. Therefore, in this limit, we find that NCOS theory also turns into commutative $\mathcal{N} = 4$ Yang-Mills theory in $(3+1)$ dimensions. We still have the S-duality relation (2.14) in this limit, where the S-duality which we have been discussing in this subsection turns out to be the ordinary electric-magnetic duality in $(3+1)$-dimensional commutative Yang-Mills theory.

2.2. OM-theory Compactified on Higher Dimensional Tori

In the previous subsection, we considered an M5-brane wrapped on a torus. In this subsection, we wrap the M5-brane on a circle of radius $R_4 = fL_4$ in the x^4 direction, in addition to the circles in the x^2 and x^3 directions.

First, we consider the electric picture of this theory, *i.e.*, we identify the x^2 direction as the '11th' direction of M-theory. As in the previous subsection, the T-duality on the circle in the x^3 direction maps the D4-brane into the D3-brane on the circles of the radii $\widetilde{R}_3$ and R_4. Here the string coupling constant is $\tilde{g}^{(2)}_{\text{str}}$ and the tension of closed strings is α'_2. After taking the T-dual on the circle in the x^4 direction, the D3-brane becomes a D2-brane

in type IIA superstring theory compactified on the circles of radii $\widetilde{R}_3$ and $\widetilde{R}_4 = \alpha'_2/R_4$. The string coupling constant becomes

$$g_{\text{str}}^{(4)} = \tilde{g}_{\text{str}}^{(2)} \left(\frac{\widetilde{R}_4}{R_4} \right)^{\frac{1}{2}} = \frac{R_2}{fL_3L_4} \sqrt{\frac{1}{2R_2 M_{\text{eff}}^3}}, \tag{2.15}$$

and the open string coupling is

$$G_4^2 = g_{\text{str}}^{(4)} f = \frac{R_2}{L_3L_4} \sqrt{\frac{1}{2R_2 M_{\text{eff}}^3}}. \tag{2.16}$$

Therefore, the OM limit (*i.e.*, $f \to 0$ with M_{eff} fixed) gives the NCOS limit in (2+1) dimensions.

D2-branes wound on the torus in the x^3 and x^4 directions are the T-dual of the wound D1-branes on the circle in the previous subsection, and the mass of the wound D2-branes equals $1/R_2$. In order to decouple the D2-branes with $\alpha'^{(2)}_{\text{eff}}$ and $\widetilde{G}_{\text{E}}$ fixed, we take the limit $R_2 \to 0$, $M_{\text{eff}} \to \infty$, and $L_3L_4 \to 0$.

In the other picture, we have a D3-brane on the circles of the radii $\widetilde{R}_2$ and R_4 with F_{45} in (2.9). Using rescaled coordinates $x'^i = fx^i$ $(i, j = 4, 5)$, we find that the closed string metric turns into $g_{ij} = \delta_{ij}$ and that the gauge field on the D3-brane becomes $A'_i = f^{-1}A_i$. Now let us contemplate the T-duality on the circle in the x^4 direction, under which the D3-brane transforms into a D2-brane. Since the T-duality maps the gauge field A'_4 to a scalar field X'^4 on the D2-brane as $A'_4 \to (1/2\pi\alpha'_3)X'^4$, (2.9) is equivalent by the T-duality to

$$\frac{f^2}{2\alpha'_3} \partial'_5 X'^4 = L_3 M_{\text{eff}}^3. \tag{2.17}$$

Using (2.8) and solving (2.17), we found that $X'^4 = f^{-1}x'^5 = x^5$. Thus, the T-duality on the circle in the x^4 direction maps the D3-brane into a D2-brane wound on a line at an angle of 45 degrees on the x'^4-x^5 plane. The radius $\widetilde{R}'_4$ of the dual circle in the x^4 direction is given by $1/\widetilde{R}'_4 = R_4/\alpha'_3 = 2M_{\text{eff}}^3 L_3 L_4$ and the string coupling constant is seen to be

$$\tilde{g}_{\text{str}}^{(4)} = \tilde{g}_{\text{str}}^{(3)} \left(\frac{\widetilde{R}'_4}{R_4} \right)^{\frac{1}{2}} = f^{\frac{1}{2}} \frac{L_3}{R_2 L_4} \sqrt{\frac{1}{2L_3 M_{\text{eff}}^3}}. \tag{2.18}$$

Then, in the OM limit, we find that (2+1)-dimensional NCOS theory is dual to the theory on the D2-brane tilted at an angle of 45 degrees on the x'^4-x^5 plane with the Yang-Mills coupling constant

$$g_{YM}^2 = \frac{\tilde{g}_{\text{str}}^{(4)}}{\sqrt{\alpha'_3}} = \frac{L_3}{R_2 L_4}. \tag{2.19}$$

7

Note that the D2-brane is spirally wrapping the cylinder in the x'^4 and x^5 directions. The distance between successive turns is finite and nonzero if we measure it by the canonically normalized scalar field. Thus, the Higgs VEV of the effective theory on the D2-brane becomes finite. Note also that, by lifting the D2-brane to M-theory, we can see that this theory is equivalent to the description of the NCOS_{2+1} made by using an M2-brane in [12], except for the fact that this theory is compactified on a 3-torus. In particular, the eleven-dimensional scale and the radius of the circle along the x^4 direction are coincident. The difference made by the compactifications may disappear in the NCOS limit since the radii of the circles are very large in the NCOS theory.

Here, we will discuss D0-branes in $(2p+1)$-dimensional NCOS theory. D0-branes have finite mass

$$M_{\text{D0}} = \frac{1}{g_{\text{str}}\sqrt{\alpha'}} = \frac{1}{G_{\text{o}}^2\sqrt{\alpha'_{\text{eff}}}}, \tag{2.20}$$

where G_{o} is the open string coupling constant in NCOS theory. From (2.20), we can see that D0-branes do not decouple from the spectrum of NCOS theory until we take G_{o} or α'_{eff} to zero. However, we can find that these D0-branes are bounded in the $D(2p)$-branes, since, if the D0-branes were outside the $D(2p)$-branes, the mass of the tachyon field in the worldvolume theory of the D0-brane would be proportional to $\alpha'^{-\frac{1}{2}}$, as shown in [11]. In the D4-branes ($p=2$) case, the bounded D0-brane corresponds to an instanton in the low-energy Yang-Mills theory. Thus, the D0-branes may play a role as a nonperturbative object in $(4+1)$-dimensional NCOS theory. In the D2-branes case, a uniform electric flux on the worldvolume corresponds to the bounded D0-branes, which preserve the same number of supersymmetries. Since the coordinates x^0 and x^1 are not compactified, any finite electric flux cannot produce a finite number of bounded D0-branes.

Now, let us move to OM-theory compactified on $(q+1)$-dimensional spatial torus ($0 \leq q \leq 3$), where the coordinates x^2 and x^j ($3 \leq j \leq q+2$) are periodically identified and their radii are R_2 and $R_j = fL_j$, respectively. By identifying R_2 as R and applying the T-duality to the q circles, we find that the OM limit becomes the $\text{NCOS}_{(4-q)+1}$ limit and that the closed string coupling constant is given by $g_{\text{str}} = G_{\text{o}}^2/f$, where

$$G_{\text{o}}^2 = \frac{R_2}{\prod_{j=3}^{q+2} L_j} \alpha'^{\frac{q-1}{2}}_{\text{eff}} \tag{2.21}$$

is the open string coupling constant.

In the $\text{NCOS}_{(1+1)}$ case, the open string coupling is $G_{\text{o}}^2 = \alpha'_{\text{eff}} R_2/(L_3 L_4 L_5)$. In order to decouple D3-branes wound on $\mathbf{T}^3$ in the x^3, x^4 and x^5 directions with mass $M_{\text{D3}} = 1/R_2$

8

and D1-branes wound on the circle in the k direction ($k = 3, 4, 5$) with mass $M_{\text{D1}} = \epsilon^{ijk} L_i L_j/(R_2 \alpha'_{\text{eff}})$, we need to take the limit $R_2, L_3, L_4, L_5 \to 0$ with $R_2/(L_3 L_4 L_5)$ fixed. In a dual picture in which R is identified as R_3, the magnetic flux F_{45} must be quantized as

$$2\pi n = \int_0^{2\pi L_4} \int_0^{2\pi L_5} dx^4 dx^5 F_{45} = 4\pi L_3 L_4 L_5 M_{\text{eff}} = 2\pi G_{\text{o}}^{-2}. \tag{2.22}$$

However, under T-duality on the circle in the x^4 or x^5 direction, the tilted D2-brane transforms into a D3-brane, not a D-string. Although we could map the D2-brane to a D-string under T-duality, instead of doing that, in the next section we will give a simple description of the S-dual theory of NCOS_{1+1} by using an M2-brane.

3. (1+1)-dimensional NCOS theory and M2-branes

In this subsection, we consider an M2-brane in M-theory compactified on a rectangular torus, which is a product of circles with circumferences $2\pi R_{11}$ and $2\pi f L_2$, with a metric $g_{\mu\nu} = \eta_{\mu\nu}$ ($\mu, \nu = 0, 1, 11$) and $g_{22} = f^2$:

$$f^2 = \frac{2M_{\text{eff}}^3}{M_{\text{p}}^3}, \tag{3.1}$$

and so the coordinate x^2 is identified under the translation by $2\pi L_2$ as $x^2 \sim x^2 + 2\pi L_2$. Let us suppose that the M2-brane worldvolume is of the form $\mathbf{R}^2 \times \mathbf{S}^1$. For the $\mathbf{R}^2$, we take coordinates x^0 and x^1. The circle $\mathbf{S}^1$ is tilted at an angle of 45 degrees on the x^2-x^{11} plane, where we assume that $L_2 = nR_{11}$ and that the M2-brane is wound once on the circle in the x^2 direction.

According to the dictionary between M-theory and type IIA theory, the scalar field X^{11} ($= 2\pi R_{11}\phi$) on the M2-brane worldvolume is the electric-magnetic dual of the gauge field in the (2+1)-dimensional DBI action of a D2-brane in type IIA theory. The scalar field ϕ has the usual normalized kinetic term and is mapped to the gauge field A as

$$\frac{\partial}{\partial x'^2}\phi = \frac{1}{2\pi g_{\text{str}} \alpha'^{\frac{1}{2}}} \frac{2\pi\alpha' F_{01}}{\sqrt{1 - (2\pi\alpha' F_{01})^2}}, \tag{3.2}$$

where we use the coordinate x'^2, which is given by rescaling x^2 as $x'^2 = fx^2$ and so is identified under the translation by $2\pi R_2 = 2\pi f L_2$. Since the circle on which the M2-brane

9

is wound tilts at an angle of 45 degrees on the x^2-x^{11} plane, *i.e.*, $\mathbf{S}^1$ on the line $x^2 = x^{11}$, we can see from (3.2) that there is nonzero background electric flux

$$2\pi\alpha' F_{01} \sim 1 - \frac{1}{2}f^2 \qquad (f \ll 1) \tag{3.3}$$

on the D2-brane worldvolume. Thus, this system of the M2-brane is equivalent to a D2-brane with $F_{01} \neq 0$ in type IIA theory with $1/\alpha' = R_{11}M_{\mathrm{p}}^3$ and $g_{\mathrm{str}}^2 = (R_{11}M_{\mathrm{p}})^3$. Thus, the OM limit $M_{\mathrm{p}} \to \infty$ is the (2+1)-dimensional NCOS limit with the open string coupling constant $G_{\mathrm{o}}^2 = \sqrt{2}(R_{11}M_{\mathrm{eff}})^{\frac{3}{2}}$ [12].

Applying T-duality to the circle in the x^2 direction, we map the D2-brane in the previous paragraph to a D1-brane with the electric flux in (3.3) in type IIB theory compactified on a circle of radius $\tilde{R}_2 = \alpha'/R_2 = f/(2nR_{11}^2 M_{\mathrm{eff}}^3)$, with the string coupling constant $\tilde{g}_{\mathrm{str}} = g_{\mathrm{str}}\sqrt{\alpha'}/R_2 = 1/(nf)$ and the effective string tension $1/\alpha'_{\mathrm{eff}} = f^2/\alpha' = 2R_{11}M_{\mathrm{eff}}^3$. As for the open string moduli, we have the noncommutative parameter $\Theta^{01} = 2\pi\alpha'_{\mathrm{eff}}$ and the open string metric $G_{\mu\nu} = f^2\eta_{\mu\nu}$ ($\mu, \nu = 0, 1$), from which we can see that the open string coupling constant is $\tilde{G}_{\mathrm{o}}^2 = f\tilde{g}_{\mathrm{str}} = 1/n$.

In the OM limit $M_{\mathrm{p}} \to \infty$ with M_{eff} fixed, this system goes to (1+1)-dimensional NCOS theory with the open string coupling constant $\tilde{G}_{\mathrm{o}}^2 = 1/n$ and the effective string tension $1/\alpha'_{\mathrm{eff}} = 2R_{11}M_{\mathrm{eff}}^3$. The mass of D-strings wound on the circle in the x^2 direction is $M_{\mathrm{D1}} = \tilde{R}_2/(\tilde{g}_s\alpha') = 1/R_{11}$. In order to decouple these D-strings with α'_{eff} fixed, we can take the limit $R_{11} \to 0$ and $M_{\mathrm{eff}} \to \infty$.

On the other hand, thinking of the x^2 direction as the '11th' direction of M-theory, we can view this system as n D2-branes with $F_{01} \neq 0$ in type IIA theory with the string tension $1/\alpha'_2 = R_2 M_{\mathrm{p}}^3 = 2nR_{11}M_{\mathrm{eff}}^3/f$ and the string coupling constant $g_{\mathrm{str}}^{(2)} = (R_2 M_{\mathrm{p}})^{\frac{3}{2}} = \sqrt{2f}(nR_{11}M_{\mathrm{eff}})^{\frac{3}{2}}$. For the $n = 1$ case, the line $x^2 = x^{11}$ on which the M2-brane is wound corresponds to nonzero electric flux on the worldvolume of the D2-brane. It is convenient to normalize the scalar field X^2 on the M2-brane so that the metric which appears in the kinetic term of the scalar fields becomes η_{AB}. Therefore, we have the relation $X^2 = fx^{11}$ (*i.e.*, $\partial_{11}X^2 = f$). As in the previous picture, by making use of

$$\frac{1}{2\pi R_2}\frac{\partial}{\partial x^{11}}X^2 = \frac{1}{2\pi g_{\mathrm{str}}^{(2)}\alpha'_2{}^{\frac{1}{2}}}\frac{2\pi\alpha'_2 F_{01}}{\sqrt{1 - (2\pi\alpha'_2 F_{01})^2}}, \tag{3.4}$$

we find that the electric flux on the D2-brane worldvolume is

$$2\pi\alpha'_2 F_{01} \sim f \qquad (f \ll 1). \tag{3.5}$$

10

For general n, the flux may be obtained as $2\pi\alpha'_2 F_{01} \sim f\mathbf{1}_n$.

Taking the T-dual on the circle in the x^{11} direction, we obtain n D1-branes with the electric flux in (3.5) in type IIB superstring theory compactified on the circle of radius $\tilde{R}_{11} = \alpha'_2/R_{11} = f/(2nR_{11}^2 M_{\text{eff}}^3) = \tilde{R}_2$ and the string coupling constant $\tilde{g}_{\text{str}}^{(2)} = g_{\text{str}}^{(2)}\sqrt{\alpha'_2}/R_{11} = nf = 1/\tilde{g}_{\text{str}}$. In the OM limit, we see that $\alpha'_2 \to 0$, $\tilde{g}_{\text{str}}^{(2)} \to 0$, and that $2\pi\alpha'_2 F_{01} \to 0$. Therefore, the OM limit can be identified as the YM limit, where this theory becomes (1+1)-dimensional $U(n)$ Yang-Mills theory with a single unit of electric flux and the coupling constant

$$\frac{1}{2\pi g_{\text{YM}}^2} = \frac{\alpha'_2}{\tilde{g}_{\text{str}}^{(2)}} = \frac{\alpha'_{\text{eff}}}{n^2}. \tag{3.6}$$

Indeed, this is the S-dualized theory of (1+1)-dimensional NCOS theory [12]. In this case, to decouple fundamental strings wound on the circle, which are the S-dual of the wound D-strings and have mass $1/R_{11}$, we have to take the same limit in the dual picture.

4. Summary and Discussion

In this paper, we have studied the compactification of OM-theory on tori, with the effective Planck scale M_{eff}. As shown in [12], depending on which circle is identified as that in the '11th' direction in M-theory – whether the direction tangent to the circle is aligned with that of nonzero H_{0ij} (i denotes the spatial direction) or not – OM-theory compactified on the circle becomes NCOS theory or NCYM theory in (4+1) dimensions. We have shown that the compactification of OM-theory on higher dimensional tori, which corresponds to the T-dual of the NCOS$_{4+1}$ on tori, gives lower dimensional NCOS theory where the effective string tension $1/\alpha'_{\text{eff}} = 2RM_{\text{eff}}^3$ and the noncommutative parameter $\Theta^{01} = 2\pi\alpha'_{\text{eff}}$.

From section 2, even after taking the NCOS limit, we still have 'T-duality' between NCOS theories. NCOS$_{(p+1)+1}$ theory from a D$(p+1)$-brane wound on a circle is mapped to NCOS$_{p+1}$ theory from an unwound Dp-brane under T-duality on the circle. Let us suppose that the circle in NCOS$_{(p+1)+1}$ theory has a coordinate s, which is identified under the translation $s \to s + 2\pi L$. Before taking the NCOS limit $f \to 0$, we assume a metric $g_{ss} = f^2$ on the circle. The T-dual of this circle is a circle with a coordinate $\tilde{s}$ identified

11

under $\tilde{s} \to \tilde{s} + 2\pi\widetilde{L}$ and with the metric $g_{ss} = f^2$. Then, the open string coupling constant G_o^2 in $\mathrm{NCOS}_{(p+1)+1}$ is mapped under the T-duality to $\widetilde{G}_o^2$ in NCOS_{p+1} as

$$\frac{\sqrt{L}}{G_o^2} = \frac{\sqrt{\widetilde{L}}}{\widetilde{G}_o^2}, \qquad \text{with} \qquad \tilde{L} = \frac{{\alpha'}_{\mathrm{eff}}}{L}. \tag{4.1}$$

By making use of (4.1), we can reproduce all the open string coupling constants G_o^2 in the NCOS theories which we studied in this paper.

In sections 2 and 3, we have demonstrated that we can understand, from the OM-theoretical point of view, what the S-duals of the NCOS theories are, although the S-dualized theories have already been identified in [12]. In particular, in (3+1) dimensions, we have seen that OM-theory compactified on a torus gives a simple heuristic derivation of the S-duality between NCOS_{3+1} and NCYM_{3+1}. When the torus is the form of a product of circles of radii R and fL, the open string coupling $\widetilde{G}_{\mathrm{E}}$ in NCOS_{3+1} is given by $\widetilde{G}_{\mathrm{E}}^2 = R/L$ and $\widetilde{G}_{\mathrm{M}}$ in NCYM_{3+1} by $\widetilde{G}_{\mathrm{M}}^2 = L/R$. Thus, the S-duality can easily be understood as the exchange $L \leftrightarrow R$. Also, it is interesting to notice that the electric flux F_{01} in the NCOS theory and the magnetic flux F_{23} in the NCYM theory are unified into the self-dual three-form field H in the M5-brane worldvolume theory.

Note added: while writing this paper, we received the paper [17], which has some overlap with ours in the discussion of S-duality between NCOS and NCYM in (3+1) dimensions. Also, upon completion of this manuscript, we received the related papers [18,19].

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